RAN-2403080104033001

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M. Sc. (Sem.-IV) Examination September - 2025

PHYSICS-Statistical Mechanics (New) (Core-PH-543)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Symbols used have their usual meaning. (3) Figures to the right indicate full marks.	Student's Signature

Q. 1. Attempt any two questions out of the (a), (b) and (c) below.

(a) (i) For an ideal Bose gas, show that **5**

$$PV = \frac{2}{3}U$$

Use : $\frac{PV}{kT} = -\sum_{\epsilon} \ln(1 - z\, e^{-\beta\epsilon})$ & $N = \sum_{\epsilon} \frac{1}{z^{-1}e^{\beta\epsilon} - 1}$

(ii) Explain how a Bose Einstein condensate can be looked upon as a mixture of two "phases." **4**

(b) (i) Obtain Planck's formula for blackbody radiation using the approach of S N Bose. **5**

(ii) Explain the variation of pressure and internal energy of an ideal Bose gas with temperature parameter T/T_c. **4**

(c) (i) What is Fermi energy? Obtain the expression for the ground state pressure of an ideal Fermi gas in terms of Fermi energy. **5**

(ii) Derive the expression for Chandrasekhar limit. What will be the scenario if a stellar object exceed the Chandrasekhar limit? **4**

Q. 2. Attempt any two questions out of the (a), (b) and (c) below.

(a) (i) Describe Ising model in one dimension. **5**

(ii) Give the idea of exchange interaction between neighboring spins and define the Heisenberg Hamiltonian. **4**

(b) (i) Describe Ising model in zeroth approximation. **5**

(ii) Explain the critical behavior of the low-field susceptibility, χ_0 of an Ising model in zeroth approximation. **4**

(c) (i) Describe Ising model in first approximation. **5**

(ii) Show that in Bethe approximation the entropy of the Ising lattice at $T = T_c$, is given by the expression **4**

$$\frac{S_c}{Nk} = \ln 2 + \frac{q}{2}\ln\left(1 - \frac{1}{q}\right) - \frac{q(q-2)}{4(q-1)} \ln\left(1 - \frac{2}{q}\right)$$

Q. 3. Attempt any two questions out of the (a), (b) and(c) below.

(a) (i) What is phase transition? Explain order of phase transition. **5**

(ii) Give the example of Phase Transition with their order parameter. **4**

(b) (i) Explain the behavior of the cluster number m_ℓ for large value of ℓ, at the condensation point. **5**

(ii) Explain Mayer's theory of phase transition. **4**

(c) (i) By considering general remarks of condensation, obtain the zeros (roots) of the grand partition function in the theory of Lee and Young. **5**

(ii) How is Lee and Yang's approach different from Mayer's approach in explaining condensation. **4**

Q. 4. Attempt any two questions out of the (a), (b) and(c) below.

(a) (i) Derive Einstein relation in the Langevin theory of Brownian motion. **4**

(ii) Using the equations $\Delta S = \left(\frac{\partial S}{\partial T}\right)_V \Delta T + \left(\frac{\partial S}{\partial V}\right)_T \Delta V = \frac{C_V}{T}\ \Delta T + \left(\frac{\partial P}{\partial T}\right)_V \Delta V$;

$\Delta P = \left(\frac{\partial P}{\partial T}\right)_V\ \Delta T + \left(\frac{\partial P}{\partial V}\right)_T\ \Delta V = \left(\frac{\partial P}{\partial T}\right)_V \Delta T - \frac{1}{K_T V} \Delta V$; and expressions

$\overline{[(\Delta T)^2} = \frac{kT^2}{C_V}$, $\overline{(\Delta V)^2} = kTK_T V$, while $\overline{(\Delta T \Delta V) = 0]}$; for $\overline{(\Delta T)^2}$, $\overline{(\Delta V)^2}$

and $\overline{(\Delta T \Delta V)}$, show that

(a) $\overline{(\Delta T \Delta S)} = kT$ (b) $\overline{(\Delta P \Delta V)} = -\,kT$

(c) $\overline{(\Delta T \Delta S)} = kT\ (\partial V/\partial T)_P$ (d) $\overline{(\Delta P \Delta T)} = kT^2 C_V^{-1}\ (\partial P/\partial T)_V$

(b) (i) Explain the essential theme of Einstein's approach for Brownian motion of particles. **4**

(ii) Derive the equations for ΔS and ΔP for equilibrium thermodynamic fluctuations. **4**

(c) (i) Consider a microscopic region V_A in the fluid and evaluate the mean square fluctuation in the number of particles, N_A, occupying this region. **4**

(ii) Derive Smoluchowski equation for Brownian motion. **4**
